XXIII. *On the Error in Standards of Linear Measure, arising from the thickness of the bar on which they are traced. By Captain* Henry Kater, *V.P. and Treasurer of the Royal Society.*

Read June 17th, 1830.

IN the course of the adjustment and verification of the copies of the Imperial standard yard, destined for the Exchequer, Guildhall, Dublin, and Edinburgh, I discovered a source of error till then, I believe, wholly unsuspected, arising from the thickness of the bar upon the surface of which measures of linear dimension are traced. The difficulties which I experienced, and the remedy which suggested itself upon that occasion, and which was found efficient, are shortly detailed in the account of the construction and adjustment of the new standards of weights and measures of the United Kingdom of Great Britain and Ireland, published in the Philosophical Transactions for 1826.

But as the notice there given occupies little more than a single page, and might therefore pass unremarked, I cannot but think that a fact of such importance in inquiries where linear measures are concerned, and which may be sufficient to account for the discrepancies to be found in the experiments of different observers, ought to be placed before the Royal Society in a more prominent point of view than that which it at present occupies. I shall, therefore, first extract from the paper alluded to the part to which I refer, and then add an account of such experiments as I have since made on the subject; and describe a scale which I have caused to be constructed so as almost entirely to obviate the source of error of which I am treating.

It may be seen in the paper before mentioned, "On the Construction of the Standard Measures," that Sir George Shuckburgh's scale was employed as the medium of comparison; the distance from the zero point of which to that marked thirty-six inches, had been found by comparisons detailed in the Phi-

losophical Transactions for 1821, not to differ sensibly from the Imperial standard yard.

The bars intended for the Exchequer, &c. were half an inch thick and one inch and a quarter wide; the thickness being nearly the same as that of Sir George Shuckburgh's scale.

The dots upon the surface of the bars having been adjusted, so that their distance appeared to be equal to the distance from zero to thirty-six inches on Sir George Shuckburgh's scale, further comparisons were made after the plugs carrying the dots had been securely fixed; "and it was with surprise and disappointment that I found the whole of them apparently too short. They had been adjusted upon a board of mahogany carefully planed; and the table upon which they were now placed was so flat as to occasion little alteration in a spirit-level passed along it. The error of the standards was, however, far too considerable to be attributed to any curvature which on this occasion could take place; and it was not until after several days that I discovered the cause of this perplexing circumstance. I found that by placing a card, the thickness of which was accurately one-fiftieth of an inch, under the middle of the standard, the distance of the dots was much increased; and by placing a card of the same thickness under each of the extremities, and withdrawing that which was under the centre, the distance of the dots was considerably diminished. The total difference amounted to no less than .0016 of an inch; whilst the double of the error, which would have arisen from mere curvature under similar circumstances, would not have been one ten-thousandth of an inch.

"The cause was now evident: by elevating the middle of the standard, the under surface was shortened and the upper surface extended; and on the contrary, when the extremities were elevated, the upper surface was compressed and the lower surface lengthened; the quantity of the effect evidently depending upon the thickness of the bar.

"Having thus assured myself of the source of the error, a method of obviating it soon presented itself. As the upper and under surfaces of the bar are in different states, the one being compressed and the other extended, there must be an intermediate plane which suffers neither extension nor compression; and this plane must be nearly midway between the two surfaces. I therefore

caused Mr. Dollond to reduce the thickness of the bar for the distance of an inch and three-quarters from its extremities to one-half; the gold disks and plugs were then inserted as before, and the adjustment completed in the manner which has been described. The plugs being secured and the projecting parts removed, the standards were repeatedly compared with Sir George Shuckburgh's scale (the standard being placed upon the scale), when no perceptible difference could be detected. Pieces of card were now placed under the standard as before, without occasioning any appreciable alteration; and I had thus experimental proof of the perfect efficiency of the remedy I had employed*."

It will now be necessary for me to state some circumstances which led to the experiments I am about to detail.

In the year 1820, I very carefully compared a scale, now in the possession of Mr. Dollond, with Sir George Shuckburgh's standard, when the distance from zero to thirty-six inches upon both scales appeared to be precisely the same. In the year 1824, I again compared Mr. Dollond's scale with Sir George Shuckburgh's, upon the occasion of examining a scale for M. Schumacher and another for M. Svanberg. Mr. Dollond's scale by these comparisons appeared to differ from Sir George Shuckburgh's only .000181 of an inch in defect. I had thus good reason to suppose the value of Mr. Dollond's scale to be well ascertained; and as it was much thinner than that of Sir George Shuckburgh's, I considered it preferable for determining the value of the scales intended for Denmark and Sweden.

Having recently received from Mr. Dollond a scale which I had instructed him to make for the Government of Russia, and also the Imperial standard yard from the House of Commons, I commenced the necessary preparations for comparing them together.

Finding by stretching a thread along the table, which I have always used for such comparisons, that its surface was concave, I had it carefully planed until the thread no longer indicated any irregularity. My first comparisons at once showed a considerable difference between the scale intended for Russia (copied from Mr. Dollond's scale) and the Imperial standard yard; at the same time that a scale which I had formerly laid off for my own use from Mr. Dollond's,

* Phil. Trans. for 1826, Part II. p. 44–46.

and which might be taken as equal to it, differed very little from the Russian scale. I now procured Sir GEORGE SHUCKBURGH's scale and Mr. DOLLOND's, and set myself carefully to investigate the cause of this difference.

It was immediately evident that Sir GEORGE SHUCKBURGH's scale still nearly agreed with the Imperial standard yard, and that there was no great difference between Mr. DOLLOND's scale, my scale, and that intended for Russia. But the difference between Mr. DOLLOND's scale and Sir GEORGE SHUCKBURGH's was so great, as to leave no doubt on my mind that in determining the value of the former, I had committed some considerable error; and as other scales are dependent upon this, it became an object of importance to ascertain its origin and amount. The origin of this error I could not for a moment hesitate in pronouncing to be the thickness of Sir GEORGE SHUCKBURGH's scale; the very great care I have been accustomed to bestow in comparisons of this kind, leaving me no other cause to which it could with probability be referred.

I have said that a thread stretched along the table indicated no irregularity of surface. I now examined the surfaces of the Imperial standard yard by stretching a thread along that surface which was perpendicular to the table. Every face of this bar was perceptibly concave; yet when laid upon the table and pressed horizontally at one end, it moved about its centre, proving that the surface of the table was convex, though the thread was not capable of indicating it.

I next procured a marble slab nearly sixty-four inches long. This I preferred in its unpolished state; as the operation of polishing being performed upon small portions of the marble in succession might destroy the plane surface procured by grinding. Upon this slab I placed the Imperial standard yard, Sir GEORGE SHUCKBURGH's scale, my own, and Mr. DOLLOND's scale; the Russian scale being laid, for want of room, upon the table. The Imperial standard yard seemed now to rest with nearly its whole surface in contact with the marble, and this, in addition to the test of the thread, I considered to be a sufficient indication that the marble was plane.

The following comparisons were then made between the different scales, in which the same microscopes and apparatus were employed as are described in an account of the comparison of various British standards of linear measure, published in the Philosophical Transactions for 1821. It is to be observed that

as the microscopes invert, an increase in the readings of the micrometer indicates that the scale is shorter than that with which it is compared, and vice versâ. The value of one division of the micrometer is .0000428742 of an inch.

Date. 1830.	Therm.	Imperial Standard Yard.	Sir George Shuckburgh's Scale.	Russian Scale.	Mr. Dollond's Scale.	Captain Kater's Scale.
		div.	div.	div.	div.	div.
May 1	66°	7	0.5	23	27.5	32.7
		8.2	2.0	23.7	25	30
2	63	104	98	120	124.5	126
		76.5	72.5	95	101	101
		8.7	5	22.5	28.5	37
	66	88	80.5	95	104	110.5
3	62	3.5	2	23	27	28.2
		3	0	24.5	26	....
		102.5	96	117	125	131
		17	14	35.5	42	46
	64	10.7	4	22.5	28.3	37
4	61	20.2	16	39	40	51
		20	16.5	37	43.6	47.5
	62	18	16	34.6	42	46
5	63	15	10.7	28	37	37.7
		14	10	26.7	33.5	42
		Means... 32.02	27.73	47.94	53.43	60.24
Readings of Imperial Stand. Yd.			32.02	32.02	32.02	32.02
Differ. from Imperial Stand. Yd.			4.29	15.92	21.41	28.22

Converting these differences of the readings of the micrometer into inches, we obtain the distances from zero to thirty-six inches on each scale in parts of the Imperial standard yard.

Sir George Shuckburgh's scale . . .	36.00018 inches
The Russian scale	35.99932 ——
Mr. Dollond's scale	35.99908 ——
Captain Kater's scale	35.99879 ——

Here it is seen that Mr. Dollond's scale, which by careful comparisons in the years 1820 and 1824 appeared to differ little, if at all, from Sir George Shuckburgh's scale, is now no less than .0011 of an inch in defect.

I now resolved to determine the amount of the error which might arise from the flexure of a bar of known thickness, the curvature of the surface upon which it is placed being given.

For this purpose I prepared sets of wires, which by careful measurement I found to be, .012, .02, .03, .04, .05, and .10 of an inch in diameter.

In order to subject the surface of the marble slab to a more rigid examination, I prepared some fine copper wire by annealing it, and then stretched it along the slab, applying as much weight as it would bear at each end without breaking. This detected a curvature of one end of the slab, which the thread did not indicate, and which was removed by placing a thick card between it and the table.

The Imperial Standard Yard.

I commenced with the Imperial standard yard. The bar upon which this yard is laid off, is 1.07 inch square.

On placing a wire of .012 of an inch diameter under the middle of the bar, the two extremities were raised above the marble; a thin wedge was placed under one extremity of the bar, the other being then in contact with the marble, in order to prevent the effect which would arise from the weight of the apparatus carrying the microscopes when placed upon the bar. This precaution, I shall content myself with remarking, was used in every instance where necessary.

The following comparisons were then made between the length of the yard when the bar lay flat upon the marble, when it was curved upwards by a wire placed under its middle, and when curved downwards by wires placed under the dots marking the yard. In the latter position the middle of the bar touched the marble.

With wires of .012 of an inch diameter.

(A.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
95.5	89.5	112.0			
96.5	91.7	111.5			
95.7	88.0	111.7			
79.0	72.5	94.5			
Mean 91.7	85.4	107.4	6.5	15.5	22.0
0.2 Reduction to the Chord.					
91.9					

As in the preceding experiments the bar was clear of the marble when a wire was placed under its middle, and touched the slab when wires were placed under its extremities; I next tried whether any greater errors would be produced by the curvature which the bar would assume by its own weight when clear of the marble in both cases. For this purpose I used wires of .05 of an inch diameter, by which the bar was wholly supported.

With wires of .05 of an inch diameter.

(B.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
67.0	60.0	79.0			
61.0	54.0	78.5			
57.5	54.0	74.5			
60.0	52.0	73.0			
57.0	55.5	74.5			
Mean 60.5	55.1	75.9	5.6	15.2	20.8
0.2 Reduction to the Chord.					
60.7					

Here it may be seen that the results are very nearly the same as in the preceding Table, and consequently that the errors of the Imperial standard yard are the greatest possible with a curvature, the versed sine of which is less than one-hundredth of an inch.

In order to obtain accurate conclusions from the above Tables, it must be considered that when the bar is lying flat upon the marble it is the length of the scale that is taken, but when the bar is curved it is the chord of the arc which is measured. It therefore becomes necessary to add to the mean of the column headed "without wires" the number of divisions of the micrometer, which is equal to the excess of the arc above its chord.

This correction being made, if the difference between the mean of the readings so corrected of the micrometer, when the bar lies flat upon the marble, and the mean of the readings when a wire is placed under the middle of the bar be taken, it will give the amount of the error arising from flexure independent of the chord when the bar is curved upwards.

In like manner the difference between such corrected mean and the mean of

the readings of the column headed "wires under the ends" will give the error proceeding from the same source when the bar is curved downwards: and the sum of the errors thus obtained will show the greatest error which can arise from a curvature, the versed sine of which is equal to the diameter of the wires employed. The same amount of errors may at once be obtained by taking the difference of the mean readings of the second and third columns.

In the Imperial standard yard it appears that with a curvature, the versed sine of which is less than .012 of an inch, the amount of the errors is .000943, or nearly one-thousandth of an inch; whilst the error which would result from the difference between the arc and its chord is absolutely insensible, not amounting to one hundred-thousandth of an inch. Now it must be obvious, that if a scale were compared at two different periods with the Imperial standard yard, the yard at one of such periods being placed upon a part of the table deviating from a plane surface .012 of an inch in a yard, and at another period the same quantity, but in a contrary direction,—the difference in the resulting values of the scale so compared would be no less than .000943 of an inch. This supposes the scale which is the subject of comparison either to be very thin, or to be placed upon a part of the table which is perfectly flat; a circumstance which it is not difficult to imagine very possible, or even that different parts of the surface of the table may be curved in contrary directions, when the small amount of the curvature in question is considered.

I may here observe, that the thickness of a single shaving which the plane takes off from the table is sufficient to occasion an error equal to that resulting from the preceding comparisons; for I found by careful measurements with a micrometer, that the mean thickness of such a shaving of deal was about .009 of an inch.

I have hitherto supposed the surface of the table not to be plane; but if the table were plane and the surface of the bar were curved, the bar would by its weight apply itself to the plane surface of the table; and a like error in either case would be the consequence.

It has been stated that the surface of the bar of the Imperial standard yard is concave; and by my present comparisons the distance from zero to thirty-six inches on Sir GEORGE SHUCKBURGH's scale, which by former measurements

appeared not to differ sensibly from the Imperial standard yard, is in excess .0001945, or nearly two ten-thousandths of an inch. Now if the surface of the marble slab be supposed to be plane, the curvature of the Imperial standard yard would have the same effect in producing error as in the case of wires being placed under its ends; and it will be seen that a curvature of the bar, the versed sine of which is only .002 of an inch in length, would be sufficient to produce the difference in question.

On examining the preceding Table, it will be perceived that the difference between the length of the yard when lying flat on the marble, and its length when curved upwards, is much less than the difference when the bar is supported at its ends, and consequently curved downwards; and hence that the neutral plane is not as I supposed in the middle of the bar, but nearer to the convex surface: but on this important part of the subject I shall have to state more hereafter.

Sir George Shuckburgh's *Scale.*

The bar upon which this scale is traced is 67.7 inches long, 1.4 inch wide, and 0.42 inch thick. The scale comprises 60 inches, the distance from the end of the bar to zero is 3.7 inches, and from 60 to the other extremity of the bar 4 inches.

The scale being placed upon the marble slab, its ends projected two inches beyond it. I could perceive light between the bar and the marble, and could pass a sheet of letter-paper under the zero end to the distance of eight inches from its extremity, that is to the fourth inch of the scale. The bar was in contact with the marble for two inches and a quarter at the other end; and from this part letter-paper could be passed between the bar and the marble for the distance of eighteen inches. The remainder of the bar rested upon the marble.

In the following experiments, the wires in elevating the extremities were placed under 0 and 60 of the scale, and under 30 when the scale was to be curved upwards.

With wires .012 of an inch diameter.

From 0 to 36 inches.

(C.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
99.0	95.5	105.0			
99.0	95.5	106.0			
Mean 99.0	95.5	105.5	3.5	6.5	10.0
0.0 Reduction to the Chord.					
99.0					

With wires .02 of an inch diameter.

(D.)

Without Wires.	A wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
6.0	3.0	14.0			
3.0	0.5	17.0			
97.0	92.0	109.3			
96.0	94.0	109.0			
Mean 50.5	47.4	62.3	3.1	11.8	14.9
0.2 Reduction to the Chord.					
50.7					

With wires .03 of an inch diameter.

(E.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
97.0	92.7	110.2			
96.5	92.0	111.0			
Mean 96.7	92.3	110.6	4.8	13.5	18.3
0.4 Reduction to the Chord.					
97.1					

With wires .04 of an inch diameter.

(F.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
16.0	7.5	32.0			
12.0	8.0	30.7			
94.5	87.0	114.0			
Mean 40.6	34.2	58.9	7.2	17.5	24.7
0.8 Reduction to the Chord.					
41.4					

Wishing to ascertain whether the curvatures of both halves of the scale were similar, I took the distance from 24 to 60 inches,

With wires .04 of an inch diameter.

From 24 to 60 inches.

(G.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
41.0	24.0	44.0			
38.0	21.0	44.0			
126.0	104.0	129.0			
Mean 68.3	49.7	72.3	19.4	3.2	22.6
0.8 Reduction to the Chord.					
69.1					

Here we perceive that the sum of the errors is very nearly the same as before; but there is an irregularity in the curvature, or rather a difference in the place of the neutral plane of the two halves of the bar, which deserves particular attention; for instead of the error being less, as in all the preceding experiments, when a wire is placed under the middle, than it is when the extremities of the bar are supported, it is here greater. We may also remark that the difference between the means of the readings "without wires" is 27.7 divisions, which shows that the distance from 24 to 60 inches of the scale is shorter than that from 0 to 36 inches .001187 of an inch.

With wires .05 of an inch diameter.

From 0 to 36 inches.

(H.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
14.5	7.5	39.5			
13.0	3.5	33.0			
11.5	4.0	31.2			
8.5	4.0	36.7			
7.0	0.0	32.0			
Mean 10.9	3.8	34.5	8.3	22.4	30.7
1.2 Reduction to the Chord.					
12.1					

With wires 0.5 of an inch diameter.

From 24 to 60 inches.

(I.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
41.5	23.0	48.0			
39.5	18.5	50.0			
39.5	19.0	49.0			
38.7	15.7	48.5			
35.0	16.0	45.0			
Mean 38.8	18.4	48.1	21.6	8.1	29.7
1.2 Reduction to the Chord.					
40.0					

From this Table the same conclusions may be drawn as from Table (G), as to the place of the neutral plane in the two halves of the bar. I am disposed to attribute this irregularity to the manner in which the scale is constructed, as it consists of a bar of cast brass, to which is soldered a piece of plate brass, upon which the divisions are traced. The difference between the distance from 24 to 60 inches and from 0 to 36 inches does not differ sensibly from that before found, namely .001187 of an inch. I should have observed that the comparisons of the distance from 0 to 36 and from 24 to 60 inches on the scale immediately followed each other, that is, the microscopes were transferred immediately from one part of the scale to the other.

Wishing to try the effect of an increased curvature, the following comparisons were made.

With wires 0.1 of an inch diameter.

From 12 to 48 inches.

(K.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
81.0	51.5	121.0			
83.5	51.0	117.0			
86.0	53.0	120.0			
83.0	51.0	120.0			
Mean 83.4	51.6	119.5	36.8	31.1	67.9
5.0 Reduction to the Chord.					
88.4					

The above Table shows that employing the middle portion of the scale, the error in either position is nearly the mean of the errors in the two preceding Tables.

Mr. DOLLOND'S *Scale.*

This scale is upon plate brass, 42 inches long, 1.6 inch wide, and 0.17 inch thick; it contains 41 inches. The scale when placed upon the marble slab appeared to be in contact with it throughout its whole extent.

The wires for elevating the ends were placed under 0 and 41 inches, and the wire when the middle was raised was under $20\frac{1}{2}$ inches.

With wires .05 of an inch diameter.

From 0 to 36 inches.

	Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
	div.	div.	div.	div.	div.	div.
	32.5	30.0	52.0			
	32.0	30.0	52.5			
	32.0	30.0	56.0			
(L.)	31.8	31.2	57.0			
	30.0	28.0	52.6			
	30.3	28.0	52.6			
	Mean 31.4	29.5	53.8	4.4	19.9	24.3
	2.5 Reduction to the Chord.					
	33.9					

Captain KATER'S *Scale.*

This scale is of cast brass, 45.2 inches long, $\frac{5}{8}$ wide, and 0.29 inch thick. The wires, when the ends were elevated, were placed under the extremities.

With wires 0.5 of an inch diameter.

From 0 to 36 inches.

	Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
	div.	div.	div.	div.	div.	div.
	105.0	97.0	137.0			
	89.0	82.5	128.0			
	92.5	80.5	123.0			
	91.0	79.0	128.0			
(M.)	95.0	80.0	128.5			
	94.0	80.0	131.0			
	Mean 94.4	83.2	129.3	13.2	32.9	46.1
	2.0 Reduction to the Chord.					
	96.4					

I shall now state the results which have been obtained from the comparisons I have detailed.

It has been already shown that a correction must be applied to the observed length of the scale, to reduce it to the chord of the given curvature. This, if the scale is only 36 inches long, may be applied at once; but with Sir George Shuckburgh's scale, the length of which is 60 inches, it becomes necessary first to find the difference between an arc of 60 inches and its chord, and then to take the proportional part of this difference for 36 inches. The same must be done with Mr. Dollond's scale, which is 41 inches long, and with my own, the length of which is 45.2 inches.

The following Table gives the difference between the arc and its chord, and the proportional part where necessary for 36 inches.

Versed Sine.	Difference of the Arc and its Chord on 36 inches.	Proportional part for 36 inches.	Equivalent Divisions of Micrometer.
inches.	inches.	inches.	
.012	.00001		
.02	.00002		
.03	.00006		
.04	.00009	....	2.1
.05	.00014		
Versed Sine.	Difference of the Arc and its Chord on 60 inches.		
.012	.000005	.000003	0.0
.02	.000013	.000008	0.2
.03	.000030	.000018	0.4
.04	.000054	.000032	0.8
.05	.000084	.000050	1.2
.10	.000400	.000240	5.0
Versed Sine.	Difference of the Arc and its Chord on 41 inches.		
.05	.000122	.000107	2.5
Versed Sine.	Difference of the Arc and its Chord on 45.2 inches.		
.05	.000112	.000089	2.0

Having applied the reduction to the chord, we obtain the error occasioned

by the extension of the surface of the bar when it is curved upwards, and the error when it is curved downwards; but in Sir GEORGE SHUCKBURGH'S scale, when a wire is placed under the middle, the surface of the bar rests upon the edges of the marble slab; the length to be considered of the bar is therefore in this position equal to the length of the marble 63.7 inches, and the error of the curvature upwards, as given in the Tables, must be increased in the proportion of 63.7 to 60.

The same method must be pursued with Mr. DOLLOND'S scale, the bar being forty-two inches long.

Lastly, these results must be reduced, but in inverse proportion, to what they would have been had the length of the scale been thirty-six inches.

As the distance from 0 to 36 inches is the part of Sir GEORGE SHUCKBURGH'S scale, which has been considered upon every occasion as equal to the Imperial standard yard, it is this portion to which all the subsequent deductions refer.

The following Table contains the results of the foregoing experiments, with each scale reduced to a bar of thirty-six inches in the manner which has been described, the bar being taken as equal in thickness to each scale respectively.

	Thickness of Bar.	Table.	Versed Sine.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
	inches.		inches.	inches.	inches.	inches.
Imperial Standard Yard	1.07	(A)	.012	.00028	.00066	.00094
Sir GEORGE SHUCKBURGH'S Scale.	0.42	(C)*	.01	.00022	.00039	.00061
		(D)	.02	.00023	.00084	.00107
		(E)	.03	.00036	.00096	.00132
		(F)	.04	.00054	.00125	.00179
		(H)	.05	.00063	.00160	.00223
Mr. DOLLOND'S Scale	0.17	(L)	.05	.00022	.00097	.00119
Captain KATER'S Scale	0.29	(M)	.05	.00071	.00176	.00247

By referring to the results in this Table, derived from Sir GEORGE SHUCKBURGH'S scale, it will be perceived that the sum of the errors increases as the versed sine or diameter of the wire employed. This will be more readily shown by dividing the sum of the errors by the corresponding diameter of the wire, or reducing each to a versed sine of .01 of an inch.

* Reduced by proportion from a versed sine of .012 to that of .01 of an inch.

Versed Sine.	Sum of Errors.	Sum of Errors Reduced to a Versed Sine of .01 of an inch.
inches.	inches.	inches.
.01	.00061	.00062
.02	.00107	.00053
.03	.00132	.00044
.04	.00179	.00045
.05	.00223	.00045
	Mean . .	.00050

The errors arising from the thickness of different scales, though nearly in proportion to their thickness, do not seem to follow any very regular law, but to depend in some measure upon the manufacture of the material employed, as well as upon the thickness of the bar: thus if we reduce the error of each scale to what it would have been had the scale been half an inch thick, thirty-six inches long, and the versed sine equal to .01 of an inch, we have the following results for comparison.

Errors resulting from a varied sine of .01 of an inch, the bar being supposed thirty-six inches long, and half an inch thick.

	Error when curved upwards.	Error when curved downwards.	Sum of Errors with a Versed Sine of .01 of an inch.
	inches.	inches.	inches.
Imperial Standard Yard	.00013	.00031	.00044
Sir George Shuckburgh's Scale. .	.00017	.00042	.00059
Mr. Dollond's Scale	.00012	.00056	.00068
Captain Kater's Scale	.00025	.00061	.00086

Here it may be seen that the sum of the errors of my scale is the greatest, and this scale is upon a bar of cast brass not hammered.

Mr. Dollond's scale is of plate brass, and Sir George Shuckburgh's appears to be made of cast brass faced with plate brass.

The Imperial standard yard is upon a bar of brass, which I should judge from its apparent hardness to have been well hammered; but the preceding result given from this bar must be too little, as it has been shown that a wire .012 of an inch diameter was more than sufficient to produce the greatest errors to which the bar is liable.

From the last Table it appears that the error of the scale when curved upwards is scarcely more than one half of its error when curved downwards; and from this it should seem that the neutral plane is not in the middle of the bar as I had supposed.

If this inference be correct, the distance of the neutral plane from that surface of the bar, the curvature of which is convex, may be found by considering the sum of the errors in the preceding Table to represent the thickness of the bar, when the corresponding error resulting from the curvature upwards will be the distance of the neutral plane from the convex surface. This distance appears to be scarcely equal to one-third of the thickness of the bar.

From the experiments which have been detailed, we are led to the following conclusions.

1. That in a standard of linear measure traced upon the surface of a bar, an error arises from the thickness of the bar when it is placed upon a table the surface of which is not plane.

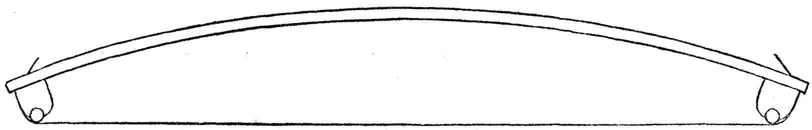
2. That this error in bars of the same material and of unequal thickness is within certain limits as the thickness of the bar, and depends upon the extension of that surface of the bar which becomes convex, and the compression of the surface which is concave.

3. That the error to which the same scale is liable from this cause, is directly as the versed sine of the curvature of the surface upon which the scale is placed.

4. That this error very far exceeds that which would arise from the difference of length between the arc and its chord under similar circumstances; so much so, that the sum of the errors from this cause in a bar one inch thick, with a versed sine of not one-hundredth of an inch, is nearly one-thousandth of an inch; whilst double the difference between the chord and the arc is not one fifty-thousandth.

It was not until I had written thus far, that a method, with which I am perfectly satisfied, occurred to me of trying a surface supposed to be plane. The difficulty, if not the impossibility of procuring what is called a straight edge is well known to workmen; but this desideratum I supplied by the very easy process of stringing a bow six feet long with piano-forte wire one-hundredth of an inch diameter, which bears a considerable degree of tension without breaking. The wire was passed over two thick wires half bedded in pieces glued

near the extremities of the bow; these served as bridges, and one was of a sufficient length to support the whole when laid upon a horizontal surface. The nature of the surface upon which it is placed may thus be examined at leisure by observing its proximity to the wire. Should the surface be convex, a wire must be placed under each extremity of the bow-string of sufficient diameter just to clear it from all but the most convex part of the surface. As this simple contrivance may be applied to a great variety of useful purposes where a straight edge is required, I trust I may be pardoned for giving the accompanying sketch of it.



By means of this apparatus I examined a variety of surfaces, any one of which I formerly should have considered as well calculated to serve as a support for scales, which were to be compared together. The following were the results.

	Length in inches.	Curvature.	Versed Sine inches.
	inches.		inches.
A mahogany dining-table	42	Concave.	0.04
A marble chimney-piece	61	Ditto	0.12
Another	62	Ditto	0.04
Another	62	Ditto	0.10
Top of a piano-forte of rosewood finely finished	48	Ditto	0.03

The front board of a piano-forte of rosewood forty-seven inches long. This was very highly finished, and the general surface was found to be nearly plane, but irregular. Here it is worthy of observation, that I could detect the nature and extent in some degree of the irregularities of the surface, by tapping with my fingers upon the wire whilst it was pressed by the weight of the bow upon the board. Where it yielded no sound, the wire was of course in contact with the surface, which in that case was either convex or plane. When the wire yielded a sound by tapping upon it with the finger, the surface was concave; and some idea might be formed of the extent, by the acuteness or gravity of the sound produced; the edges of the concavity serving as bridges, which limited

the length of the string. So delicate is this test by sound, that a concavity can be detected by this method where the interval between the wire and the surface under examination is not to be perceived by the eye.

Having now a means superior to any I before possessed of examining a plane surface, I applied it to the marble slab, and detected a slight concavity not exceeding one-hundredth of an inch. This was corrected by placing cards under different parts of the marble. When I had adjusted it so that no irregularity was perceptible, I thought it necessary to repeat my former comparisons.

The marble most carefully made plane by the bow.

Date. 1830.	Imperial Standard Yard.	Sir George Shuckburgh's Scale.	Russian Scale.	Mr. Dollond's Scale.
	div.	div.	div.	div.
May 31	7	5.5	26	31.0
	4.5	6.2	25	33.0
	4.7	7.0	24	33.2
	4.7	4.8	23.2	32
	3.3	3.3	20	30
June 1	4.0	6.0	22	31.2
	7.0	4.0	22	30.0
	5.0	4.0	21	32
2	10	6.0	23.5	31.3
	8	3.0	25	31
	6.3	3.0	22	32.5
	6	4.5	21.2	30
	3	3.5	23.5	31
	7	4	21.5	30.5
3	98.0	97	114.5	122.3
	97.5	96.5	115	123.5
4	28	22	43	53
	27.5	20	40.5	51
5	38.0	33.5	53	59.5
	37.3	31	50	59.3
Mean	20.34	18.24	36.79	45.36
Readgs of Impl. Stand. Yd		20.34	20.34	20.34
Diff. from Impl Stand. Yd		2.10	16.45	25.02

From these differences converted into inches, we have now the following distances from 0 to 36 inches on each scale in parts of the Imperial standard yard.

Sir George Shuckburgh's scale . . . 36.00009 inches.
The Russian scale . . . . 35.99929 ——
Mr. Dollond's scale . . . . 35.99893 ——

These results differ but little from those of the former comparisons, to which, however, I think they are to be preferred.

It has been shown that from the present results, the value which has been hitherto given to Mr. DOLLOND's scale requires a considerable correction. Mr. DOLLOND's scale, by former measurements, appeared to be equal to 35.99991 inches of Sir GEORGE SHUCKBURGH's scale, and consequently to be shorter than that scale only .00009 of an inch; but it now appears to be shorter than Sir GEORGE SHUCKBURGH's scale .00116 of an inch. Sir GEORGE SHUCKBURGH's scale, therefore, when the former comparisons were made, must have been curved downwards in consequence of that part of the surface of the table upon which it was placed being concave; and we may remark that by consulting the preceding Tables, it will be seen that a curvature, the versed sine of which is .03 of an inch in a yard, would have been sufficient to occasion the error in question.

With reference to this error in the former estimation of the value of Mr. DOLLOND's scale, it is important to add that in the year 1828 I employed it (not having then access to Sir GEORGE SHUCKBURGH's standard) in determining the value of a scale for the Government of Hanover. This scale was made by Mr. DOLLOND, according to the mode which will hereafter be described; and when referred to Sir GEORGE SHUCKBURGH's scale, then taken as equivalent to the Imperial standard yard, it appeared to be equal to 35.99973 inches: upon that occasion the scale for Hanover was found by numerous comparisons to be shorter than Mr. DOLLOND's scale .00018 of an inch, which being subtracted from 35.99893 inches, the last determination of the value of Mr. DOLLOND's scale, leaves for the true length of the Hanoverian scale 35.99875 inches of the Imperial standard yard.

Having now shown the nature and magnitude of the error, which is the subject of this paper, I shall proceed to point out the means of obviating it.

It has been seen that the error in question results from the extension and compression of the surfaces of the bar upon which the scale is laid off dependent upon its curvature, and that there is a neutral surface which suffers neither extension nor compression, and which appears from the preceding experiments to be at about one-third of the thickness of the bar from that surface which becomes convex. When it is the object to have two points only on the

bar, marking for example the yard, it has been already shown that by cutting away one-half of the thickness of the bar at its ends and placing the points upon the new surface, the error which arises from flexure is reduced to the least possible quantity; as this (the difference between one-half and one-third of the thickness of the bar) is the nearest approach that can be made to the neutral surface when the bar is curved upwards and when it is curved downwards.

When a scale of inches is required, this method is not available; as the whole surface of the bar must in that case be employed. But by diminishing the thickness of the bar, the magnitude of the error is diminished proportionably; and it is evident that the thickness of the bar might be so reduced as to render the error scarcely appreciable.

Having prepared a bar, or rather a plate, of a thickness which is no more than sufficient to receive the divisions of the scale, and to preserve an even surface, the next object is to provide this plate with a proper support. For this purpose a bar of brass well hammered is to be employed, of a sufficient width, and half an inch or even an inch thick. Upon the surface of this bar the thin plate intended to receive the divisions is to be placed, and made to slide freely in a dovetailed groove, formed by two side-plates of similar thickness, screwed to the surface of the bar. Lastly, the thin plate upon which the scale is to be laid off is to be fixed at its middle point to the bar by a single screw passing through it.

Now if the thick bar which forms the support be curved, its surfaces will suffer extension and compression in proportion to the thickness of the bar. The thin plate accommodating itself to the curvature of the bar will follow the same law, and the resulting error will be less in the proportion of the thickness of the plate to that of the bar. Now it has been shown that the greatest errors to which the Imperial standard yard (one inch thick) is liable, amount to nearly one-thousandth of an inch; as a wire of one-hundredth of an inch diameter placed under the middle of the bar is more than sufficient to produce the greatest curvature of which it is susceptible; and it follows that if such a bar were to form the support of a plate of one-tenth of an inch in thickness in the manner just described, the sum of the greatest errors, to which a scale

so constructed would be liable from curvature, could not exceed one ten-thousandth of an inch.

The support of the scale which I have caused to be made for the Government of Russia, and the comparisons of which, with the Imperial standard yard, are given in the present paper, is a brass bar one inch and three-quarters wide, and half an inch thick. The plate carrying the divisions is one-tenth of an inch thick; but were I to construct another, it should not exceed half this thickness. This plate is made to slide freely upon the surface of the bar in a dovetailed groove, and is fixed to the bar at its middle point by a single screw in the manner before described. The divisions are thirty-six inches, marked by dots upon the surfaces of gold pins let into the brass plate. An additional inch beyond zero is divided in like manner into tenths of an inch. This description will serve also for that of the scale before alluded to, made for the Government of Hanover.

The following experiments were made with the Russian scale, the arrangements being the same as before detailed.

Russian scale with wires .04 of an inch diameter.
From 0 to 36 inches.

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
69	66	75			
71	66	73.5			
71	65	74			
70	67.5	75			
70	63.5	75			
70	64	73.5			
Mean 70.5	65.3	74.3	7.3	1.7	9.0
2.1 Reduction to the Chord.					
72.6					

I now employed wires of .05 of an inch in diameter, and these supported the bar wholly above the surface of the marble.

With wires .05 of an inch diameter.
From 0 to 36 inches.

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
38.5	35.5	43.0			
39.5	32.5	39.5			
36.2	31.5	39.0			
Mean 38.1	33.2	40.5	7.0	0.3	7.3
2.1 Reduction to the Chord.					
40.2					

From these last Tables it appears that the sum of the greatest errors to which the Russian scale is liable from flexure is the greatest possible when the versed sine is .04 of an inch, and is then equal to nine divisions of the micrometer, or .00038 of an inch, which, reduced as in the foregoing experiments with the other scales to a versed sine of .01 of an inch, gives for the comparative sum of the errors due to such versed sine .00009, or about one ten-thousandth of an inch.

It is evident that the advantage of this construction consists merely in the employment of a thin plate to receive the divisions, instead of a bar of greater thickness; but a plate sufficiently thin could not well be used without a firm support, which is necessary to protect it from injury, as well as to prevent the plate from being affected by any sudden variation of temperature, either from partial currents of air, or from the vicinity of the person of the observer.